Descriptional Complexity of Pushdown Store Languages

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Descriptional complexity: questions

Take the length of description as complexity measure.

- How succinctly can a model represent a formal language in comparison with other models?
- What is the maximum blow-up when changing from one model to another? (Upper bounds)
- Are there languages such that a maximum blow-up is achieved? (Lower bounds)

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Results

- Recursive trade-offs
- Non-recursive trade-offs

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- ...
- Finite automata and the size of their syntactic monoid (Holzer, König 2002)

Pushdown store languages

The pushdown store language of a PDA M is the set $P(M)$ of all words occurring on the pushdown store along accepting computations of M.

$$P(M) = \{u \in \Gamma^* \mid \exists x, y \in \Sigma^*,\ q \in Q,\ f \in F :$$
$$(q_0, xy, Z_0) \vdash^* (q, y, u) \vdash^* (f, \lambda, \gamma),\ \text{for some } \gamma \in \Gamma^*\}.$$

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Theorem (Greibach 1967)

Let M be a PDA. Then, $P(M)$ is a regular language.

Example

The language $\{\, a^n b^n \mid n \geq 1 \,\}$ is accepted by the following (deterministic) PDA

$$M = \langle \{q_0, q_1, q_2\}, \{a, b\}, \{Z, Z_0\}, \delta, q_0, Z_0, \{q_2\} \rangle$$

such that

$$\begin{aligned} \delta(q_0, a, Z_0) = \{(q_0, ZZ_0)\}, \;\; & \delta(q_0, a, Z) = \{(q_0, ZZ)\}, \\ \delta(q_0, b, Z) = \{(q_1, \lambda)\}, & \\ \delta(q_1, b, Z) = \{(q_1, \lambda)\}, \;\; & \delta(q_1, \lambda, Z_0) = \{(q_2, Z_0)\}. \end{aligned}$$

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The pushdown store language is $P(M) = Z^* Z_0$.

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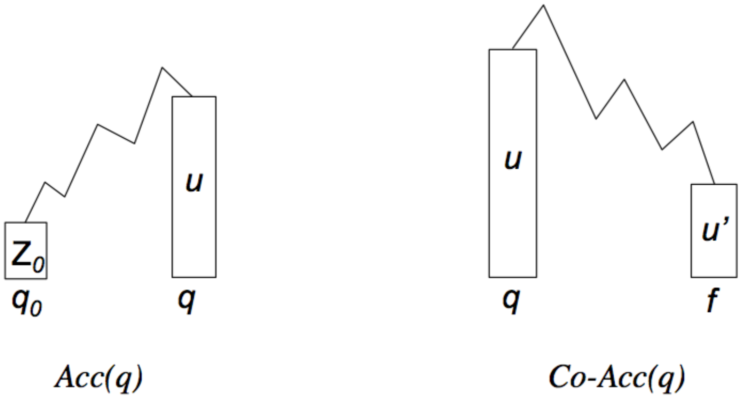
$$\mathit{Acc}(q) = \{u \in \Gamma^* \mid \exists x, y \in \Sigma^* : (q_0, xy, Z_0) \vdash^* (q, y, u)\},$$
$$\mathit{Co\text{-}Acc}(q) = \{u \in \Gamma^* \mid \exists y \in \Sigma^*, f \in F, u' \in \Gamma^* : (q, y, u) \vdash^* (f, \lambda, u')\}.$$



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- The union over all $q \in Q$ gives a factor $|Q|$.
- Altogether, we need $|Q|^3|\Gamma| + |Q|^2(|\Gamma| + 1) + |Q| + 1$ states.

Finite automata construction improved

Avoid the union (factor $|Q|$) by considering

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- The removal of the first symbol is for free.
- Altogether, we need $|Q|^2(|\Gamma| + 1) + |Q|(2|\Gamma| + 3) + 2$ states.

Lower bounds

Consider the language family $L_{m,k}$ for $m \geq 2$ and $k \geq 1$:

$$L_{m,k} = \{(a^{m^2} b^{m^2})^{(k-1)/2} a^{m^2} c\}, \text{ for odd } k,$$
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$L_{m,k}$ can be accepted by a PDA with $O(m)$ states and $O(k)$ pushdown symbols whereas every NFA for $P(L_{m,k})$ needs at least $\Omega(m^2 \cdot k)$ states.

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Theorem

Let $M = \langle Q, \Sigma, \Gamma, \delta, q_0, Z_0, F \rangle$ be a PDA. Then, an NFA for $P(M)$ exists with $O(|Q|^2|\Gamma|)$ states. On the other hand, there exist infinitely many PDA $M_{Q,\Gamma}$ of size $O(|Q| \cdot |\Gamma|)$ such that every NFA accepting $P(M_{Q,\Gamma})$ needs $\Omega(|Q|^2|\Gamma|)$ states.

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- Observe that $P(M) = \{u \in \Gamma^* \mid u \in Acc(q) \text{ and } q \in F\}$.

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Lemma

For $m, k \geq 2$, there exist PDA $M_{m,k}$ which can never pop having m states and k pushdown symbols, for which every NFA for $P(M_{m,k})$ needs at least $k \cdot m + 1$ states.

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Lemma

For any $k \geq 0$, there exists a stateless PDA M_k having $|\Gamma_k| = k + 1$ pushdown symbols, for which every NFA for $P(M_k)$ needs at least $k + 2 = |\Gamma_k| + 1$ states.

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Lemma

Let M be a counter PDA with state set Q. Then, $P(M)$ is accepted by some NFA with size bounded by $|Q| + 2$. Moreover, this size bound is optimal.

Applications: complexity of decidability questions

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- For the construction of the set $Acc(Q)$, the reachability of $O(|Q|^2|\Gamma|^2)$ pairs has to be tested.
- Each test can be seen as an instance of the emptiness problem for context-free languages which is in P.
- An NFA for $Acc(Q)$ can be constructed in deterministic polynomial time.
- Similarly, an NFA for $Co\text{-}Acc(Q)$ can be constructed in deterministic polynomial time as well as for the intersection of both and the removal of the first symbol.

Applications: complexity of decidability questions

Lemma

Given a PDA M, it is P-complete to decide whether:
(i) $P(M)$ is a finite set. (ii) $P(M)$ is a finite set of words having at most length k, for a given $k \geq 1$.

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Corollary

Given an unambiguous PDA M, it is P-complete to decide whether: (i) M is a constant height PDA. (ii) M is a PDA of constant height k, for a given $k \geq 1$.

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Corollary

Given a PDA M, it is P-complete to decide whether M is essentially a counter machine.

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- Some decidability questions are solvable in P and P-hard.

- Consider other special cases, e.g., m-counter PDA or turn-bounded PDA.
- Investigate trade-offs occurring when determinizing the NFA for $P(M)$.

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- Tight bounds of $\Theta(|Q|^2|\Gamma|)$ in the general case.
- Better and also tight bounds for special cases.
- Some decidability questions are solvable in P and P-hard.

- Consider other special cases, e.g., m-counter PDA or turn-bounded PDA.
- Investigate trade-offs occurring when determinizing the NFA for $P(M)$.
- Extend the decidability of being a constant height PDA to arbitrary PDA.